Professional Development Module

Title: Teaching Fractions in Grades 3 - 6

Content and Instructional Shifts: K-5

Targeted Audience: Teachers in grades 3-6

Grade Span: 3-6

Description: Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Delivery time: Session 7 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

Session 7 Instructor Notes:

Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics.*
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

Success Criteria:

- Teachers will describe how to use Multiple Groups problems to develop student understanding of addition and subtraction of fractions.
- Teachers will classify Partial Groups problems as multiplication, partitive division, or measurement division.
- Teachers will use contextual problems to begin to make sense of multiplication and division of fractions.

Time: 3 hours

Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Partial Groups Problem Situations for Multiplication"
- Handout "Partial Groups Problem Situations for Division"
- Instructor Resource "Samples of Student Work for Addition"
- Student work collected by each participant

Session 7 Activity 1
Analyze Student Work from Implementation Assignment 6

Approximate Time: 30 minutes
Key Purpose: To reflect on teacher actions during the last implementation assignment.
Materials:

- Student work collected by each participant

Activity Description	Key Discussion Points
Analyze Lessons on Fraction Equivalency and Order Have participants work with a partner. Have each participants share the following for the Equal Sharing problem he or she posed to students: • What number combinations did you use? • What equivalent fractions resulted in student work? • How did your class discussion address equivalency? Have each participants share the following for the Equivalencing problem, Open Number Sentence or Comparison problem he or she posed to students: • What did you notice about your students' thinking? • What did you discuss? • Was the discussion similar or different from the Equal Sharing problem discussion? • What did you learn?	**Analyze Lessons on Fraction Equivalency and Order** The purpose of this activity is for teachers to discuss what they did to develop students' understanding of fraction equivalence and order. As participants work in groups, note examples of evidence of students' understanding of fraction relationships. Ask select teachers to share their student's work and thinking with the entire class.

Session 7 Activity 2
Addition and Subtraction of Fractions

Approximate Time: 30 minutes
Key Purpose: To understand the importance of students developing intuitive strategies before learning the standardized procedures for adding and subtracting fractions.
Materials:

- Instructor Resource "Samples of Student Work for Addition"

Activity Description	Key Discussion Points
1. Multiple Groups problems Show "Samples of Student Work for Addition" (instructor resource). This resource includes student work for four different Multiple Groups problems. As you share the work, pose the following questions:	**1. Multiple Groups problems** **Problem 1:** If it takes $\frac{3}{4}$ yard of ribbon to make a bow, how many yards of ribbon will it take to make 8 bows? • This is a Multiple Groups multiplication problem. The student work

• What type of problem is this (Equal Sharing, Multiple Groups multiplication or Multiple Groups measurement division)? • Why did the problem lead to addition of fractions with like or unlike denominators? • What strategy did the students use? • How might a teacher use the student work to help the entire class make sense of addition of fractions?	is from a 3rd grade class. • Students who represent each group or use Grouping and Combining Strategies often solve Multiple Groups multiplication problems by adding fractions with like denominators. • The first student used Repeated Addition. This student shows an understanding of adding fractions with like denominators, decomposing $\frac{3}{4}$ into $\frac{2}{4}+\frac{1}{4}$, and the commutative and associative properties of addition. For example, $\frac{3}{4}+\frac{3}{4}+\ldots+\frac{3}{4}=(\frac{2}{4}+\frac{1}{4})+(\frac{2}{4}+\frac{1}{4})+\ldots+(\frac{2}{4}+\frac{1}{4})=(\frac{2}{4}+\frac{2}{4})+\ldots+(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4})=6$. Standard 4.NF.B.3b addresses decomposing fractions. The second student used Repeated Addition and shows an understanding of how to add fractions and mixed numbers with like denominators. • The teacher might have these students share their work with the class and discuss why the notation and results make sense. **Problem 2:** Eight children want to share 3 pies so they each have the same amount of pie and all the pie is gone. How much pie should each child get? • This is an Equal Sharing problem. The student work is from a 4th grade class. • Students who use Sharing One Item at a Time often add fractions with like denominators. Students who use Non-anticipatory Sharing or who use Additive Coordination Sharing Groups of Items often add fractions with unlike denominators or give an answer as a combination of two fractions, such as $\frac{1}{4}$ and $\frac{1}{8}$. • The first student used Additive Coordination Sharing One Item at a time and shows an understanding of adding unit fractions with like denominators. The second and third students use Additive Coordination Sharing Groups of Items. Both students give their answer as the sum of two fractions with unlike denominators. • This student work provides an opportunity for teachers to ask the class to determine whether or not $(\frac{1}{4}+\frac{1}{8})$ and $\frac{3}{8}$ represent the same amount. **Problem 3:** Twelve children in art class have to share 8 packages of

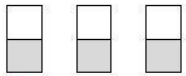
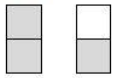
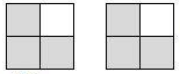
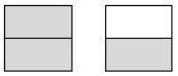
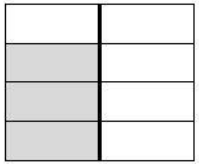
	clay so that everyone gets the same amount. How much clay can each child have? • This is an Equal Sharing problem. The student work is from a 5th grade class. • Students who use Non-anticipatory Sharing or who use Additive Coordination Sharing Groups of Items often add fractions with unlike denominators or give an answer as a combination of two fractions. • The first student used Additive Coordination Sharing Groups of Items and shows an understanding of adding unit fractions with like denominators. The second and third students used Non-anticipatory sharing or Additive Coordination Sharing Groups of Items. It is difficult to know which strategy without talking to the student. In each case his or her answer is a combination of two fractions with unlike denominators, $\frac{1}{2}$ and $\frac{1}{6}$. • This student work provides an opportunity for the teacher to ask the class to determine whether or not $\frac{1}{2}+\frac{1}{6}$ and $\frac{2}{3}$ represent the same amount. The second and third students' visual model clearly show $\frac{1}{2}=\frac{3}{6}$. The teacher might use these models to facilitate the discussion. **Problem 4:** There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally? • This is an Equal Sharing problem. The student work is from a 5th grade class. • Equal Sharing problems can lead to adding fractions with like and unlike denominators. • It appears the first, third, and fourth students used Non-anticipatory Sharing or Additive Coordination Sharing Groups of Items. It is difficult to know without talking to the student. The first student correctly adds $\frac{1}{2}+\frac{1}{4}$. The second student used Additive coordination Sharing One Item at a Time. This results in adding unit fractions with a common denominator. The third and fourth students seem to add denominators and get an incorrect answer.

	• This student work provides an opportunity for the teacher to discuss how to add fractions with unlike denominators, $\frac{1}{2}+\frac{1}{4}$. This discussion might be prompted by asking which answer is correct.
2. Classroom Experiences Have small groups discuss the following questions: • Have you discussed fraction addition or subtraction as a result of students getting different answers for Multiple Groups problems? • What prompted the discussion? Share the specific problem and student answers. • What numbers did you use in the problem? • What evidence of student understanding resulted from the class discussion?	**2. Classroom Experiences** This discussion might occur when studying the previous student work. If time is short, you might skip this activity. Have several teachers share classroom experiences on adding fractions as a result of posing Multiple Groups problems. Note to the instructor: You will address addition and subtraction of fractions again in Session 8.

Session 7 Activity 3
Multiplication of Fractions

Approximate Time: 40 minutes
Key Purpose: To develop an understanding of fraction multiplication.
Materials:
- Handout "Partial Groups Problem Situations for Multiplication"

Activity Description	Key Discussion Points
1. "Partial Groups Problem Situations for Multiplication" Question 1 Have the participants complete "Partial Groups Problem Situations for Multiplication" (handout) question 1 in small groups. Discuss the similarities and differences among the problems as a whole class. Solve each problem with direct modeling. Describe your reasoning. a. A punch recipe calls for 2 cups of sugar. How much sugar do I need to triple the batch? b. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar do I need to triple the batch? c. A punch recipe calls for 2 cups of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch? d. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch?	**1. Partial Groups Problem Situations for Multiplication" Question 1** a. A way to model this problem is to show 3 groups of 2.  It is important to emphasize the convention in textbooks in the United States is that the first factor represents the number of groups and the second factor represents the size of the groups in a multiplication grouping problem. b. A way to model this problem is to show 3 groups of $\frac{1}{2}$. There are two common ways to do this.

	 3 groups of $\frac{1}{2}$ cup is $1\frac{1}{2}$ cups OR  3 groups of $\frac{1}{2}$ cup is $1\frac{1}{2}$ cups c. A way to model this problem is to show $\frac{3}{4}$ of a group of 2. There are two common ways to do this.  $\frac{3}{4}$ group of 1 cup + $\frac{3}{4}$ group of 1 cup is $1\frac{1}{2}$ cups OR  $\frac{3}{4}$ group of 2 cups is $1\frac{1}{2}$ cups d. A way to model this problem is to show $\frac{3}{4}$ of $\frac{1}{2}$.  $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$ Similarities: The four problems are multiplication situations. You know the number of groups and the size of each group. You can multiply to find the total. Difference: The first two problems have a whole number of groups, while the last two problems have a fraction of a group.
2. "Partial Groups Problem Situations for Multiplication" Question 2 Have the participants complete the table shown in question 2 in small groups. Discuss the results as a whole class.	**2. "Partial Groups Problem Situations for Multiplication" Question 2** The following table shows correct responses for question 2. Note the last two problems are Partial Groups problems. A Partial Groups problem is one in which the number of groups is not a whole number. A Multiple Groups problem is one in which there is a whole number of groups and a fractional amount in each group where the fraction is not equal to a whole number.

Word Problem	Number of Groups	Amount per Group	Total	Possible Equation	Problem Type
A punch recipe calls for 2 cups of sugar. How much sugar do I need to triple the batch?	3	2	?	3 x 2 = ?	Whole Number Multiplication
A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar do I need to triple the batch?	3	$\frac{1}{2}$	?	3 x $\frac{1}{2}$ = ?	Multiple Groups Multiplication
A punch recipe calls for 2 cups of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch?	$\frac{3}{4}$	2	?	$\frac{3}{4}$ x 2 = ?	Partial Groups Multiplication
A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch?	$\frac{3}{4}$	$\frac{1}{2}$	?	$\frac{3}{4}$ x $\frac{1}{2}$ = ?	Partial Groups Multiplication

3. "Partial Groups Problem Situations for Multiplication" Question 3

Have participants complete question 3 in small groups. Discuss the results as a whole class.

Solve the last two problems again using relational thinking. Write an equation to make your relational thinking explicit.

a. A punch recipe calls for 2 cups of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch?

b. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar do I need to make $\frac{3}{4}$ of a batch?

3. "Partial Groups Problem Situations for Multiplication" Question 3

a. You can think of this problem as finding $\frac{3}{4}$ of a group of 2.

$\frac{1}{4}$ of 2 is $\frac{1}{2}$, so $\frac{3}{4}$ of 2 is 3 x $\frac{1}{2}$ or $1\frac{1}{2}$

This reasoning uses the associative property of multiplication.

$\frac{3}{4}$ x 2 = (3 x $\frac{1}{4}$) x 2 = 3 x ($\frac{1}{4}$ x 2)

b. You can think of this problem as finding $\frac{3}{4}$ of a group of $\frac{1}{2}$.

$\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$, so $\frac{3}{4}$ of $\frac{1}{2}$ is 3 x $\frac{1}{8}$ or $\frac{3}{8}$

This reasoning uses the associative property of Multiplication.

$\frac{3}{4}$ x $\frac{1}{2}$ = (3 x $\frac{1}{4}$) x $\frac{1}{2}$ = 3 x ($\frac{1}{4}$ x $\frac{1}{2}$) = 3 x $\frac{1}{8}$ = $\frac{3}{8}$

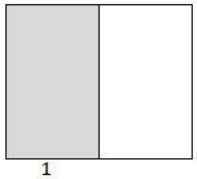
4. "Partial Groups Problem Situations for Multiplication" Question 4

Have the participants complete question 4 in small groups. Discuss the results as a whole class.

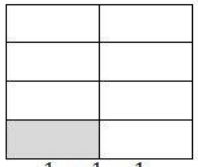
How might you use the last problem to make sense of the traditional algorithm for multiplying fractions?

4. "Partial Groups Problem Situations for Multiplication" Question 4

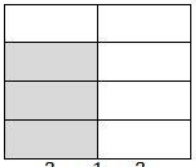
The traditional algorithm is $\frac{3}{4}$ x $\frac{1}{2}$ = $\frac{3 \times 1}{4 \times 2}$ = $\frac{3}{8}$. To make sense of this algorithm ask the following questions: What is 1 fourth of $\frac{1}{2}$? (Multiply by $\frac{1}{4}$ x $\frac{1}{2}$ to get $\frac{1}{8}$). If 1 fourth of $\frac{1}{2}$ is $\frac{1}{8}$, what is 3 fourths of $\frac{1}{2}$? (Multiply by 3). The following diagram may help make sense of the algorithm.



$\frac{1}{2}$



$\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$

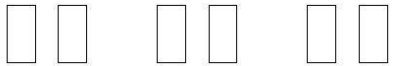
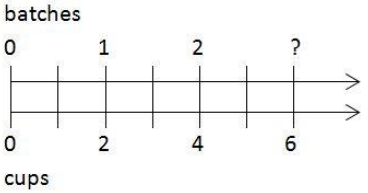
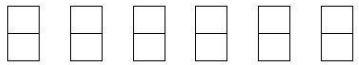


$\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$

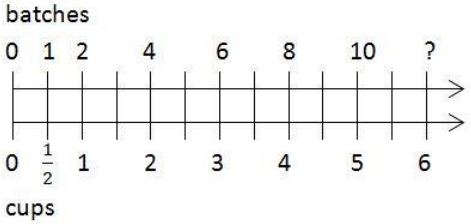
Session 7 Activity 4
Division of Fractions

Approximate Time: 70 minutes
Key Purpose: To develop understanding of fraction division.
Materials:

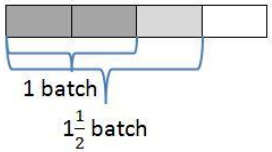
- Handout "Partial Groups Problem Situations for Division"

Activity Description	Key Discussion Points
1. " Partial Groups Problem Situations for Division " Question 1 Have the participants complete "Partial Groups Problem Situations for Division" (handout) question 1 in small groups. Discuss the similarities and differences among the problems as a whole class. Solve each problem with direct modeling. Describe your reasoning. a. A punch recipe calls for 2 cups of sugar. How many batches can I make with 6 cups of sugar? b. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How many batches can I make with 6 cups of sugar? c. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How many batches can I make with $\frac{3}{4}$ cup of sugar? d. A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much of a batch can I make with $\frac{3}{8}$ cup of sugar?	**1. " Partial Groups Problem Situations for Division " Question 1** This activity addresses two equally important ideas. One idea is for teachers to recognize the difference between measurement division and partitive division problems. A second idea is to recognize the difference between Multiple Groups problems and Partial Groups problems. All four problems under question 1 are measurement division. There are a variety of ways to model each problem. Teachers who are familiar with double number lines, may use this model for these problems. a. One way to model this problem is to show how many groups of 2 are in 6.  6 cups is equal to 3 groups of 2 cups, so 3 batches A second way to model this problem is to use a double number line. The top number line represents the number of batches and the bottom number line represents the number of cups.  b. One way to model this problem is to show how many halves are in 6.  6 cups is equal to 12 half-cups, so 12 batches A second way to model this problem is to use a double number

line.

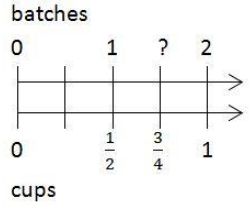


c. One way to model this problem is to show how many halves are in 3 fourths. The rectangular region is equal to 1 cup.

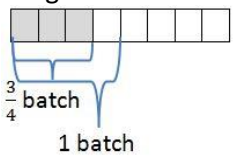


$\frac{3}{4}$ cup is equal to $1\frac{1}{2}$ batches

A second way to model this problem is to use a double number line.

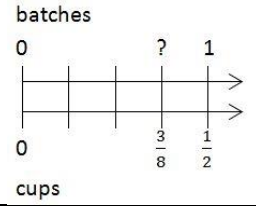


d. One way to model this problem is to show how much of 1 half is in 3 eighths. The rectangular region equals 1 cup.



$\frac{3}{8}$ cup is equal to $\frac{3}{4}$ of a half-cup, so $\frac{3}{4}$ batch

A second way to model this problem is to use a double number line.



Similarities: The four problems are measurement division situations. You know the total and size of each group. You can divide to find the number of groups.

Differences: The first two problems result in a whole number of groups, while the last two problems result in a fraction of a group.

2. " Partial Groups Problem Situations for Division" Question 2

Have the participants complete the table shown in problem 2 in small groups. Discuss the results as a whole class.

2. " Partial Groups Problem Situations for Division " Question 2

The following table shows correct responses for problem 2. Note the last two problems are Partial Groups problems.

Word Problem	Number of Groups	Amount per Group	Total	Possible Equation(s)	Problem Type
A punch recipe calls for 2 cups of sugar. How many batches can I make with 6 cups of sugar?	?	2	6	? x 2 = 6 6 ÷ 2 = ?	Whole Number Measurement Division
A punch recipe calls for $\frac{1}{2}$ cup of sugar. How many batches can I make with 6 cups of sugar?	?	$\frac{1}{2}$	6	? x $\frac{1}{2}$ = 6 6 ÷ $\frac{1}{2}$ = ?	Multiple Groups Measurement Division
A punch recipe calls for $\frac{1}{2}$ cup of sugar. How many batches can I make with $\frac{3}{4}$ cup of sugar?	?	$\frac{1}{2}$	$\frac{3}{4}$	? x $\frac{1}{2}$ = $\frac{3}{4}$ $\frac{3}{4}$ ÷ $\frac{1}{2}$ = ?	Partial Groups Measurement Division
A punch recipe calls for $\frac{1}{2}$ cup of sugar. How much of a batch can I make with $\frac{3}{8}$ cup of sugar?	?	$\frac{1}{2}$	$\frac{3}{8}$	? x $\frac{1}{2}$ = $\frac{3}{8}$ $\frac{3}{8}$ ÷ $\frac{1}{2}$ = ?	Partial Groups Measurement Division

3. " Partial Groups Problem Situations for Division " Question 3

Have the participants complete "Partial Groups Problem Situations for Division" (handout) problem 3 in small groups. Discuss the similarities and differences among the problems as a whole class.

Solve each problem with direct modeling. Describe your reasoning.

a. I have 6 cups of sugar. I have enough sugar to make a double batch of punch. How much sugar is needed for one batch?

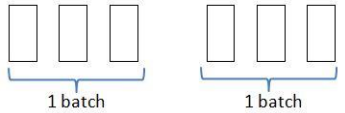
b. I have 6 cups of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of punch. How much sugar is needed for a full batch?

c. I have $\frac{3}{8}$ cup of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of

3. "Partial Groups Problem Situations for Division" Question 3

All four problems are partitive division. Again, there are a variety of ways to model each problem. Most teachers will not use a double number line unless they are already familiar with the model.

a. One way to model this problem is to show how much is in one group or batch.



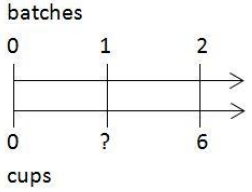
6 cups is equal to 2 batches, so 3 cups is equal to 1 batch

A second way to model this problem is to use a double number line. Notice the same thinking for the diagram shown above

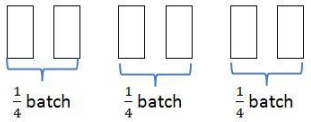
punch. How much sugar is needed for a full batch?

d. I have $\frac{1}{2}$ cup of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of punch. How much sugar is needed for a full batch?

applies to the double number line (6 cups is equal to 2 batches, so 3 cups is equal to 1 batch).

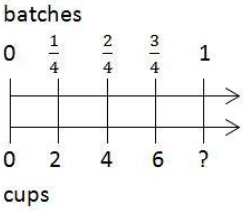


b. One way to model this problem is to show how much is in one group or batch.

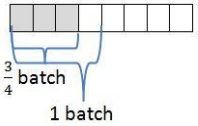


2 cups is equal to $\frac{1}{4}$ batch, so 8 cups is equal to 1 batch

A second way to model this problem is to use a double number line. Again the same thinking applies to the double number line.

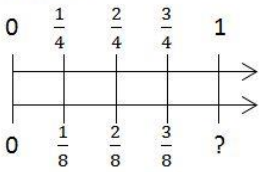
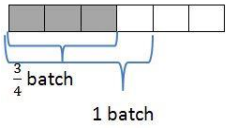
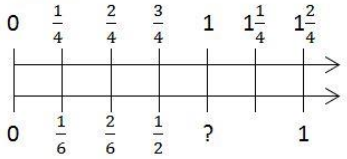


c. One way to model this problem is to show how much is in one group or batch. The rectangular region equals 1 cup. Another way to model this is to make the rectangular region equal 1 batch.



$\frac{3}{8}$ cup is equal to $\frac{3}{4}$ batch, so $\frac{1}{8}$ cup is equal to $\frac{1}{4}$ batch and $\frac{4}{8}$ or $\frac{1}{2}$ cup is equal to 1 batch

A second way to model this problem is to use a double number line. The same thinking applies to the double number line.

	batches  cups d. One way to model this problem is to show how much is in one group or batch. Again the rectangular region equals 1 cup.  $\frac{1}{2}$ cup is equal to $\frac{3}{4}$ batch, so $\frac{1}{6}$ cup is equal to $\frac{1}{4}$ batch and $\frac{4}{6}$ or $\frac{2}{3}$ cup is equal to 1 batch A second way to model this problem is to use a double number line. The same thinking applies to this model. Showing 1 cup on the bottom number line may help identify $\frac{4}{6}$ cup or $\frac{2}{3}$ cup is needed for 1 batch. batches  cups
4. "Partial Groups Problem Situations for Division" Question 4 Have the participants complete the table shown in problem 4 in small groups. Discuss the results as a whole class.	**4. "Partial Groups Problem Situations for Division" Question 4** The following table shows correct responses for problem 4. Note the last three problems are Partial Groups problems.

Word Problem	Number of Groups	Amount per Group	Total	Possible Equation(s)	Problem Type
I have 6 cups of sugar. I have enough sugar to make a double batch of punch. How much sugar is needed for one batch?	2	?	6	$2 \times ? = 6$ $6 \div 2 = ?$	Whole Number Partitive Division
I have 6 cups of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of punch. How much sugar is needed for a full batch?	$\frac{3}{4}$	?	6	$\frac{3}{4} \times ? = 6$ $6 \div \frac{3}{4} = ?$	Partial Groups Partitive Division
I have $\frac{3}{8}$ cup of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of punch. How much sugar is needed for a full batch?	$\frac{3}{4}$	?	$\frac{3}{8}$	$\frac{3}{4} \times ? = \frac{3}{8}$ $\frac{3}{8} \div \frac{3}{4} = ?$	Partial Groups Partitive Division
I have $\frac{1}{2}$ cup of sugar. I have enough sugar to make $\frac{3}{4}$ of a batch of punch. How much sugar is needed for a full batch?	$\frac{3}{4}$	?	$\frac{1}{2}$	$\frac{3}{4} \times ? = \frac{1}{2}$ $\frac{1}{2} \div \frac{3}{4} = ?$	Partial Groups Partitive Division

Session 7 Activity 5
Assignment

Approximate Time: 10 minutes

Materials:

- Handout "Session 7 Assignment Sheet"

Activity Description	Key Discussion Points
1. Reading Assignment: • *Extending Children's Mathematics*, Chapter 8 (pp. 178-208) 2. Discussion Question: • Complete question 5 on "Partial Groups Problem Situations for Division" (handout) and be prepared to discuss your thoughts during session 8.	This assignment is different from past assignments. It includes a reading assignment, but not an implementation assignment. Chapter 8 is challenging for many teachers, so encourage participants to give themselves plenty of time to study the chapter. Participants will have an implementation assignment on fraction computation after session 8.